18. Eliminate the arbitrary function 'f' from $z = f\left(\frac{xy}{z}\right)$.

19. If $\vec{F} = xyz\vec{i} + xyz^2\vec{j} + x^2yz\vec{k}$ then find div curl $\vec{F}$.

20. Find the Fourier series for the function $f(x) = x^2$ in $-\pi \le x \le \pi$.

APRIL/MAY 2024

23UEEL22 — BASIC MATHEMATICS

Time : Three hours Maximum : 75 marks

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SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. If $x = r\cos\theta$, $y = r\sin\theta$ then find $\frac{\partial(r,\theta)}{\partial(x,y)}$.

2. Write the statement of Leibnitz theorem.

3. Solve $(D^2 - 9)y = 0$.

4. Solve $(D^2 + D + 1)y = e^x$.

5. Form the partial differential equation by eliminating the arbitrary constants from $Z = (a + x)(y + b)$.

6. Form partial differential equation by eliminating 'f' from $z = f\left(\frac{y}{x}\right)$.

7. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, find $\nabla \times \vec{r}$.

8. If $\phi(x,y,z) = x^2y + y^2x + z^2$ find $\nabla\phi$.

9. Evaluate $\int \log x \, dx$.

10. Write the formulas for the fourier coefficients 'an' and 'bn'.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

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11. (a) Find the n^{th} derivative of $\frac{1}{x^2+5x+6}$.

Or

(b) If $u = x^2 + y^2 + z^2$, $v = x + y + z$ and $w = xy + yz + zx$ then show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 0$.

12. (a) Solve $(D^2 - 4D + 4)y = e^{2x}$.

Or

(b) Solve $(D^2 + 4D + 4)y = e^{-2x} + \sin x$.

13. (a) Form a partial differential equation by eliminating the arbitrary function from $f(x+y+z, xyz) = 0$.

Or

(b) Eliminate the arbitrary function 'f' from $z = e^y f(x+y)$.

14. (a) Prove that $\nabla \cdot [\nabla r^n] = n(n+1)\, r^{n-2}$.

Or

(b) If $\nabla\phi = (y^2 - 2xyz^3)\vec{i} + (3 + 2xy - x^2z^3)\vec{j} + (6z^3 - 3x^2yz^2)\vec{k}$ find ϕ.

15. (a) Evaluate $\int x \sin 2x \, dx$.

Or

(b) Find the Fourier series for the function $f(x) = e^x$ in $(-\pi, \pi)$.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. If $y = (x + \sqrt{x^2+1})^m$ show that $(x^2+1)y_{n+2} + (2n+1)x\, y_{n+1} + (n^2 - m^2)y_n = 0$.

17. Solve $(D^2 + 6D + 8)y = e^{-2x} + \cos^2 x$.